MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(All sections / Groups)

17 OCTOBER 2019
9:00 am – 11:00 am
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 8 printed pages (inclusive of the front page) with 4 questions only. Pages 6 to 8 are appendices for the rules of inference, logical equivalence laws and Standard Normal distribution table.
2. Attempt **ALL FOUR** questions. The distribution of marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.
4. Show **ALL** of your working steps clearly.

QUESTION 1 (10 MARKS)

a) Let $A = \{1,2,3,4,5,6\}$, $B = \{1,3,5,7\}$ and $C = \{1,2,4,6,8\}$

i) Find $A - B$, $B - A$ and $A \Delta B$. [2 marks]

ii) Find $P(B - A)$. [1 mark]

b) Given R and S be relations on $A = \{1,2,3\}$ that $R=\{(1,1),(1,2),(2,1),(2,2),(3,1),(3,3)\}$ and $S=\{(1,3),(1,2)\}$.

i) Is R reflexive? Justify your answer. [1 mark]

ii) Is R symmetric? Justify your answer. [1 mark]

iii) Is R transitive? Justify your answer. [1 mark]

iv) Find $S \circ R$ and $R \circ S$. [2 marks]

c) Let $R = \{ (x,y) \mid x > 2y \}$ be the relation on the set of integers.

i) List at least 2 elements of R. [1 mark]

ii) Is R a function? Justify your answer. [1 mark]

QUESTION 2 (15 MARKS)

a) Use equivalence laws to show that $(p \wedge q) \rightarrow (r \wedge q)$ and $p \rightarrow \neg q \vee r$ are logically equivalent. Please justify each step with appropriate equivalence laws. [3.5 marks]

b) Given the following arguments:

Premises:
$\exists x(R(x) \wedge S(x))$, $\forall x(\neg Q(x) \rightarrow P(x))$, $\forall x(Q(x) \rightarrow \neg R(x))$, $\forall x(\neg S(x))$
Conclusion:
$\exists x P(x)$

Use **rules of inference** to show that the arguments are true. Please state clearly each of the rules that you used in your proof. [6 marks]

c) Given the recurrence relation as follows,

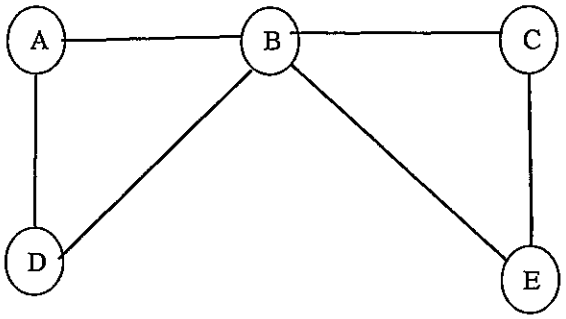
$$a_0 = 1, \quad a_1 = 11, \quad a_n = 4a_{n-1} + 5a_{n-2}, \quad n \geq 2.$$

Use strong mathematical induction to prove that $a_n = (-1)^{n+1} + 2(5^n)$ for every integer $n \geq 0$, is the explicit formula for the above recurrence relation. [5.5 marks]

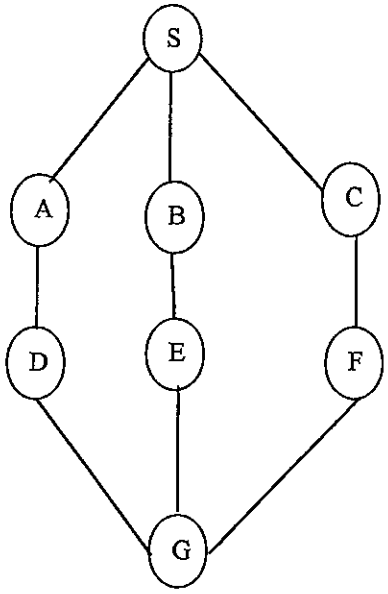
Continued ...

QUESTION 3 (10 MARKS)

a) Determine whether the following graph has a(n) Euler circuit. Construct a circuit if one exists. [2 marks]



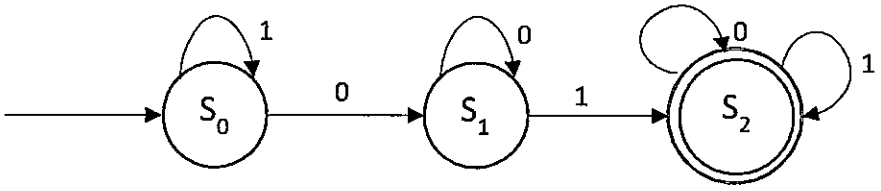
b) Given the following graph:



Use the Breadth First Search (BFS) algorithm to find a spanning tree in the above graph. Assume **vertex S** is the root and the vertices are ordered alphabetically. Show clearly each step of how the algorithm is performed and present your answer in a table with three columns, the first column is the number of steps, the second column is the stack for the search algorithm and the third column is the edges of the resulting spanning tree. [5 marks]

Continued ...

c)



Consider the finite-state automaton given above, with s_0 and s_2 to be the initial and final state respectively.

i) To what state does the automaton go when the symbols of the following strings are input to it in sequence, starting from the initial state?

1110001 , 0001000 and 11110000

[1 mark]

ii) Which of the strings in part (i) sends the automaton to an accepting state?

[1 mark]

iii) What is the language accepted by the automaton? [1 mark]

QUESTION 4 (15 MARKS)

[**Important:** Calculators are not allowed in questions 4(a) and 4(b). You must show proper steps on how to simplify and compute your answers.]

a) How many ways can a coach choose three swimmers among five swimmers?

[1 mark]

b) Given the following word,

B E A U T Y

i) In how many ways can the letters of the word be arranged? [1 mark]

ii) If the vowels are always together, how many arrangements are possible?

[2 marks]

Continued ...

c) Two different suppliers, Supplier A and Supplier B, provide a manufacturer with the same kind of battery for a particular smart phone. All supplies of this battery are kept in a large bin.

In the past, 5% of the batteries supplied by Supplier A and 9% of the batteries supplied by Supplier B have been defective. Supplier A supplies four times as many batteries as Supplier B.

Suppose you reach into the bin and select a battery, and find that it is non-defective, what is the probability that it was supplied by Supplier A?

[4 marks]

d) At a certain housing estate, it is found that, in a lawn which has been treated with weed killer, dandelion plants occur at an average rate of 0.1 per m^2. In a lawn which has not been treated with weed killer, dandelion plants occur at an average rate of 0.3 per m^2.

You may assume that the number of dandelion plants in a lawn may be modelled by a Poisson distribution.

Mr Brown has a rectangular lawn measuring 7 m by 3 m. He uses weed killer to treat his lawn.

Mrs Green has a rectangular lawn measuring 5 m by 4 m. She does not use weed killer on her lawn.

Find the probability that:

i) Mr Brown's lawn has exactly 4 dandelion plants. [2 marks]

ii) Mrs Green's lawn has at least 6 but fewer than 10 dandelion plants.

[3 marks]

iii) Mr Brown's lawn has at least one dandelion plant and Mrs Green's lawn has none. [2 marks]

End of Questions.

Appendix I

List of Rules of Inference

ADD : $p \Rightarrow (p \vee q)$

SIMP : $[p \wedge q] \Rightarrow p$

CONJ : $p \wedge q \Rightarrow (p \wedge q)$

MP : $[(p \rightarrow q) \wedge p] \Rightarrow q$

MT : $[(p \rightarrow q) \wedge (\neg q)] \Rightarrow \neg p$

HS : $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$

DS : $[(p \vee q) \wedge (\neg p)] \Rightarrow q$

RES : $[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$

List of Logical Equivalence Laws

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Conversion of Equivalence: $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

Double Negation: $\neg \neg p \Leftrightarrow p$

DeMorgan	:	(i)	$\neg (p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$		
		(ii)	$\neg (p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$		
Domination	:	(i)	$p \wedge F \Leftrightarrow F$	(ii)	$p \vee T \Leftrightarrow T$
Negation	:	(i)	$p \wedge \neg p \Leftrightarrow F$	(ii)	$p \vee \neg p \Leftrightarrow T$
Identity	:	(i)	$p \wedge T \Leftrightarrow p$	(ii)	$p \vee F \Leftrightarrow p$
Commutative	:	(i)	$p \wedge q \Leftrightarrow q \wedge p$	(ii)	$p \vee q \Leftrightarrow q \vee p$
Idempotent	:	(i)	$p \vee p \Leftrightarrow p$	(ii)	$p \wedge p \Leftrightarrow p$
Distributive	:	(i)	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$		
		(ii)	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$		
Associative	:	(i)	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$		
		(ii)	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$		
Absorption	:	(i)	$p \vee (p \wedge q) \Leftrightarrow p$		
		(ii)	$p \wedge (p \vee q) \Leftrightarrow p$		

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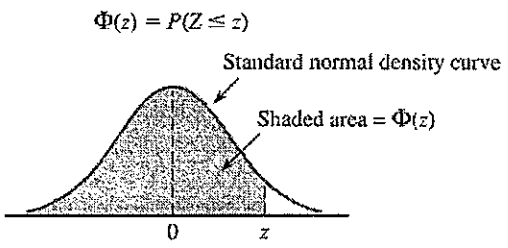
Appendix II Standard Normal Curve Areas

Standard Normal Curve Areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Continued ...

Standard Normal Curve Areas

$\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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